## Problem 3-1(a)

Given a difference equation, the problem asks us to determine the output sequence, the pulse-transfer function of the system, and the poles and zeros of the given system. To solve the problem, we first have to use z-transform to determine the pulse-transfer function of the system.

The difference equation is given as in Equation (1):

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|  | (1) |

We then get Equation (2) by applying z transform to Equation (1).

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|  | (2) |

By dividing the right hand side of the equation with the left hand side gives us Equation (3), we get the pulse transfer function of the system .

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| --- | --- |
|  | (3) |

By re-ordering the denominator of Equation (3), we can get Equation (4).

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| --- | --- |
|  | (4) |

The poles of the system are the values of z that make the denominator zero, while the zeros are the values of z that make the numerator zero.

From Equation (4), it is trivial that the zeros of the system are 0. As for the poles, we have to obtain the values of z that make the denominator zero.

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|  | (5) |

We first let . This results in z equal to 1. As for , the value of z is 0.5.

From above, we can see that the system has zeros at 0 and poles at 1 and 0.5, with the system transfer function being Equation (6).

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|  | (6) |

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## Problem 3-1(b)

To determine the output sequence of the difference equation, we have to take the initial values provided in the problem into account.

For the right hand side, the z-transform of  is equal to . As for the z transform of , it should be re-written as Equation (7):

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|  | (7) |

The z transform for  should be re-written as Equation (8):

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|  | (8) |

The z transform for  is . By combining all of the z transforms, we can get Equation (9) as follows:

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|  | (9) |

We can then get Equation (10) by re-ordering the left-hand side of Equation (9):

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|  | (10) |

By moving  to the right-hand side and dividing both sides by ,  can then be expressed as follows in Equation (11):

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|  | (11) |

By re-ordering the right-hand side of Equation (11),  can be re-written as Equation (12) and Equation (13).

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|  | (12) |

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|  | (13) |

The right-hand side of Equation (13) can be broken down into the sum of three parts as shown in Equation (14):

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|  | (14) |

By reducing the fractions on the right hand side of Equation (14) to a common denominator, we can get Equation (15):

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|  | (15) |

By expanding the coefficients in the numerator, the right-hand side of Equation (15) can be re-written as Equation (16)

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|  | (16) |

Since Equation (13) and Equation (16) are equal, we can solve the values of a, b, and c by Equations (17) to Equations (19).

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|  | (17) |
|  | (18) |
|  | (19) |

The values of a, b, and c can then be solved as , , and . Thus, the right-hand side of Equation (14) can be written as Equation (20).

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|  | (20) |

Combining Equation (13), Equation (14), and Equation (20),  can be written as Equation (21).

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|  | (21) |

The output sequence, indicated in Equation (22), can then be solved according to Equation (21).

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|  | (22) |

By re-ordering Equation (22), we arrive to the final result that is shown in Equation (23).

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|  | (23) |

## Problem 3-2

This problem is set to simulate the results of a continuous-time system with a fixed damping coefficient and given ranges for a and b. Zero and pole placements are the major factors that determine the simulation results. The overshoot for each pair of value is provided in the simulation results.

In the following simulations, the damping coefficient of the continuous-time system is set as 0.7. In the equation , the range for a and b are set as  individually. The simulations are done by ticking b at 0.25 each time and a at 0.15. So, let us first have a look at the results in Figure 1.3-1 to Figure 1.3-7, when a is fixed at 0.3 and the values of b ranging from -0.75 to 0.75.

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| Figure 1.3-1. The step response and the control signal for a = 0.3 and b = -0.75. The overshoot of the system is 0.046. |

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| Figure 1.3-2. The step response and the control signal for a = 0.3 and b = -0.5. The overshoot of the system is 0.046. |

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| Figure 1.3-3. The step response and the control signal for a = 0.3 and b = -0.25. The overshoot of the system is 0.046. |

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| Figure 1.3-4. The step response and the control signal for a = 0.3 and b = 0. The overshoot of the system is 0.046. |

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| Figure 1.3-5. The step response and the control signal for a = 0.3 and b = 0.25. The overshoot of the system is 0.046. |

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| Figure 1.3-6. The step response and the control signal for a = 0.3 and b = 0. The overshoot of the system is 0.046. |

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| Figure 1.3-7. The step response and the control signal for a = 0.3 and b = 0.75. The overshoot of the system is 0.046. |

We can see that when a is fixed and b is moved from -0.75 to 0.75, the overshoot of the system does not change. The major difference takes place after 50.

We then have a look at the results when b is fixed at -0.25 and a moving from -0.3 to 0.3. the results are shown in Figure 1.3-8 to Figure 1.3-12.

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| Figure 1.3-8. The step response and the control signal for a = -0.3 and b = -0.2. The overshoot of the system is 22.6. |

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| Figure 1.3-9. The step response and the control signal for a = -0.15 and b = -0.25. The overshoot of the system is 22.72. |

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| Figure 1.3-10. The step response and the control signal for a = 0 and b = -0.25. The overshoot of the system is 0. |

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| Figure 1.3-11. The step response and the control signal for a = 0.15 and b = -0.25. The overshoot of the system is 1.046. |

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| Figure 1.3-12. The step response and the control signal for a = 0.3 and b = -0.25. The overshoot of the system is 1.046. |

We can see that when b is fixed at -0.25 and a moving from -0.3 to 0.3, the overshoot of the system changes largely. When a is set as -0.15, the overshoot is the largest, with a value of 22.72. However, when a is set at 0, the overshoot becomes zero as well. For a with values larger than zero, the overshoot values are rather smaller compared to a with values lower than zero. We can see that for both a values that are larger than zero, the overshoot values are the same, but the response time differs.

## Problem 3-3

The main issue of this problem is to find the magnitude of the spectrum­­­ of a signal that is sampled at different rates.

The horizontal axis refers to the frequency of the signal, while the vertical axis refers to the magnitude of the signal. The given signal is a triangular wave, with magnitude one and frequency of 20 rad/s. It is also known that . F

In part (a), the sampling time is . Which means that (rad/s). From the graph given in the problem, we can roughly sketch 3 triangle waves as in Figure 1.4-1:

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| Figure 1.4-1. Three triangle waves are produced, each of magnitude 1 and a period of 20 rad/s. Aliasing occurs as the sampling time is shorter than the period of each triangular wave. The overlapped regions all have summed magnitude of one. |

Since the period for each cycle is longer than the sampling rate, we see that aliasing occurs between adjacent triangular waves. At , we can see that the peak of the second wave meets with the lowest value, which is zero, of the first and third waves. At , we can see that both the second and the third wave have values at 0.5, which means the sum of the value at that point is equal to one. Similarly, for each value of , the sum of the values are equal to one. So, by summing up the overlapped areas, we get the magnitude of the spectrum as in Figure 1.4-2:

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| Figure 1.4-2. The aliased result of the magnitude of the spectrum for part (a). |

In part (b), the sampling time is . This means that (rad/s), which is equal to the period of each triangular wave provided in the problem set. Thus, we can know that the start and end point of adjacent triangular waves meet at value 0. The peak value of the triangular waves is one and situated at  that are multiples of 20. This sampling frequency is also known as the Nyquist Frequency. We can then roughly sketch the magnitude of the spectrum as below in Fig 1.4-3:

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| Figure 1.4-3. Three triangle waves are produced, each of magnitude 1 and a period of 20 rad/s. Since the sampling distance is equal to the period of each triangular wave, the start and end point of adjacent waves meet at 0. The peak value of the triangular waves is one and situated at  that are multiples of 20. |

As In part (c), the sampling time is . This means that (rad/s), which is way larger than the period of each triangular wave. Thus, adjacent triangular waves are separated, with the peak value being one and situated at  that are multiples of 50. We can then roughly sketch the magnitude of the spectrum as below in Fig 1.4-4:

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| Figure 1.4-4. Three triangle waves are produced, each of magnitude 1 and a period of 20 rad/s. Since the sampling distance is much larger than the period of each triangular wave, adjacent waves are far apart from each other, with the peak value being one and situated at  that are multiples of 50. |

From the above results, we can see how sampling rates effect the magnitude of the spectrum. If the sampling rate is too high, aliasing occurs, resulting in a change in the magnitude of the spectrum. As for lower sampling rates, effects will not be visible.

## Problem 3-4

This problem is aimed to understand the key information of a plenary speech given by Yutaka Yamamoto.

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| **Plenary Speech** | Signal Processing via Sampled-Data Control  - A Challenge to Go Beyond Shannon [1] |
| **Speaker** | Yutaka Yamamoto [1] |
| **Location** | Hyderabad, India [1] |
| **Date** | August 2013 [1] |
| **Key Information** | * Difference between control and signal processing: feedback * Shannon Model🡪Crude signal model, large ringing effect * Problems that need to be solved * Loss of high frequency components * Mathematical formula is not causal * Slow convergence * Sharp cut-off characteristics * Problems with digital signal processing: * Not time invariant system * No transfer function * No steady-state response * No frequency response * Proposed Solution: Lifting * Up sampling and Down sampling * Interpolator virtually doesn’t have ringing * YY filter restores high frequency components * Applications: * Music Apps * Images: Solve mosquito noise, Better interpolation results * Comparison between Unser’s Method and Proposed Method * Unser’s Method: consistent sampling * Proposed Method produces better results |
| **Brief Summary** | * Using the proposed sampled-data control theory to gain better signal processing results * Proposed method minimizes error frequency response [2] * Propose an analog signal model that optimizes the intersample behavior |

References

[1] Yutaka Yamamoto, T. (2013, August). Lunch and learn: Signal Processing via Sampled-Data Control - A Challenge to Go Beyond Shannon [Video file]. Retrieved from http://www.ieeecss-oll.org/lecture/signal-processing-sampled-data-control-challenge-go-beyond-shannon

[2] Yutaka Yamamoto. (2011, May). Sampled-data Control and

Signal Processing – Beyond the Shannon Paradigm. [Online]. Available